GEOMETRY OF CONTACT AND TECHNOLOGICAL ASPECTS OF RELIEVING OF GEAR CUTTING WORM HOBS

Aleksandr Sandler[1] [ORCID: 0000-0003-2197-0447]

[1] *L.L.C. "SELLAKTION", Open Joint Stock Company "Stankoagregat", Self-employed researcher, Moscow, Russia*

Abstract: *Contact line geometry at relieving of tooth flanks of gear cutting hobs and functional relations for calculation of relieving machine setting parameters for achieving the required accuracy of the tooth profile of the generating rack are determined. They include the setting angle of the grinding wheel axis and profiles of grinding wheel active surfaces for each of the tooth flanks. A method of calculating the single setting angle of the grinding wheel axis for both tooth flanks of the hob is developed.*

Key words: radial relieving, tooth flanks of hobs, relieving machine, grinding wheel profiling, involute helical surface, curvature radii, curvature, contact lines, double-sided relieving

1. INTRODUCTION

The basis of the generating worm of a hob at cutting spur and helical gears of the involute meshing is the generating rack of the hob which is a set of cutting edges of teeth in one groove of the hob. When cutting the teeth of spur and helical gears, the generating worm of a cutting tool on a gear hobbing machine is formed by positioning the axis of the helical surface of tooth cutting edges in such a way that the generating rack in the machining area is parallel to the normal cross section (or perpendicular to the direction) of the gearwheel teeth to be cut. The helical surface of the hob on which the cutting edges of the hob teeth are located is the generating surface of the hob. The relieved surfaces of the hob flanks which form rear angles during cutting are analogs of helical surfaces with helix angles one of which is increased for one flank relative to the helix angle of the hob generating surface thread, and decreased for the other flank.

The nomenclature of hobs and deviations from the straightness of the cutting edges profile of the hob teeth are specified by the valid standards ISO 2490:2007 “Solid (monobloc) gear hobs with tenon drive or axial keyway, 0.5 to 40 module – Nominal dimensions” and ISO 4468:2020 “Gear hobs - Accuracy requirements”. These standards only apply to single-thread precision hobs.

In the technological process of hob production, relieving of flanks with a grinding wheel is still the most time-consuming operation.

The author's works on relieving the teeth of gear-cutting hobs in accordance with the mentioned standards were related to determining the principal possibility of achieving the required profile accuracy for each of the hob flanks with parallel axes of the disk grinding wheel and the hob. However, when machining the flank with a smaller helix angle of the relieved surface, the parallel arrangement of the grinding wheel and hob axes leads to a significant limitation of the grinding wheel diameter due to undercutting the upper part of the tooth following the machined one.

In the practice of tool production of hobs, a method of machining each of the flanks with the setting angle of the grinding wheel axis equal to the helix angle of the generating surface of the hob on the pitch cylinder has been established. At the same time, the geometry of generation of contact lines of the grinding wheel and the relieved surfaces has not been functionally determined until now. In order to obtain the required accuracy of tooth cutting edge profile, a long adjustment of the relieving machine by the setting and profiling parameters of the grinding wheel is carried out; trial grinding passes and intermediate control operations are performed. Thus, the problems of relieving the tooth flanks of precision hobs for spur and helical gears remain relevant up to the present time [1 – 5].

The aim of the present work is, first of all, to clarify the geometry of formation of contact lines of grinding wheel surfaces and relieved surfaces during the operation of relieving the flanks. On the basis of this research for the operation of radial (the most widespread type) relieving we define functional relations for the calculation of the basic parameters of relieving machine setting to achieve the required accuracy of the tooth profile of the generating rack: the inclination angle of the grinding wheel axis relative to the horizontal plane and profiles of the grinding wheel active surfaces for machining each of the flanks. In particular, we define the method of calculating a single angle of the grinding wheel axis, so that the machine resetting for machining the second flank consisted practically only in replacing the grinding wheel (on the faceplate) with the appropriate active surface on the grinding spindle of the machine.

2. BASIC PARAMETERS OF HOB FLANKS

Parameters of the ground surface are determined on the basis of the initial data: module m_0 of the generating rack (teeth of the cut gearwheel), pressure angle $\alpha_0 = 20°$ of the generating rack profile, helix angle γ_{m0} on its pitch cylinder and axial angle α_{0x} of the profile of the generating surface of the hob:

$$\mathrm{tg}\,\alpha_{0x} = \mathrm{tg}\,\alpha_0 \,/\, \cos\gamma_{m0} \quad (1)$$

The value of the angle γ_{m0} is determined from the known relation:

$$\sin\gamma_{m0} = Z\,m_0 \,/\, d_{m0} \quad (2)$$

where d_{m0} is the pitch cylinder diameter. For a new hob you can accept $d_{m0} = d_{a0} - 2{,}5\,m_0$ where d_{a0} is the standardized value of the hob external diameter.
To get the required profile axial angle α_{0x} of the generating surface of the hob with helical chip grooves (tooth faces), the relieved surface of teeth should have the profile angle α_x defined taking into account the lead P_z of the chip groove, the number of threads Z of the generating surface, the decline K of the relieved surface on one angular pitch of the hob teeth. For right side hobs, the values of angles α_x for the right ($_R$) and left ($_L$) flanks are determined by the formulas [6 – 8]:

$$\alpha_{xR} = \alpha_{0x} + 57{,}296\,k\,\sin^2\alpha_{0x} \,/\, [P + p + k\,\mathrm{tg}\,\alpha_{0x}];\quad \alpha_{xL} = \alpha_{0x} - 57{,}296\,k\,\sin^2\alpha_{0x} \,/\, [P + p - k\,\mathrm{tg}\,\alpha_{0x}] \quad (3)$$

where $P = P_z / 2\pi$ is the helical parameter of the face surface, P_z is the lead of the chip groove (helical face surface); $p = 0{,}5\, m_0 Z / \cos\gamma_{m0}$ is the helical parameter of the generating surface of the hob, $k = K\,z_0 / 2\pi$ is the parameter of relieving, K is the decline of the tooth back assigned in the direction of relieving movement (for radial relieving - perpendicular to the hob axis); z_0 is the number of chip grooves of the hob (or teeth in its cross section); 57,296 is the coefficient of transition from the radian to degree value of the angular correction.
Thus, for a right-handed hob the axial angle α_{xR} of the right flank profile is greater than the axial angle α_{0x} of the generating surface profile of the hob, the angle α_{xL} of the left flank profile of teeth is less than the angle α_{0x}. For left-hand hobs, it is the reverse order: $\alpha_{xL} > \alpha_{0x} > \alpha_{xR}$.
Calculated values of the back decline K for hob versions 1 and 2 are given in Table 1 with the accuracy of setting of K value to 0,1 mm. On some models of used relieving machines, the scale division value of setting K is up to 0,5 mm. Therefore, in Table 1, in addition to the calculated values, the recommended values of the decline K of the relieved surface are shown in brackets, rounding the values to 0,5 mm.

Table 1: Values of decline K of the tooth back at the angular pitch of the hob (rounding to 0,5 mm)

Version 1				Version 2					
m_0, mm	**0,5…1,0**	**0,5…1,125**	**1,25…2,0**	**1,0…1,375**	**1,5…1,75**	**2,0…2,25**	**2,5…2,75**	**3,0**	**3,5**
d_{a0}, mm	24	32	40	50	55	65	70	75	80
z_0	12	12	10	14	14	14	14	14	14
K, mm	1,3 (1,5)	1,7 (2)	2,5	2,3 (2,5)	2,5	3,0	3,1 (3,0)	3,4 (3,5)	3,6 (3,5)
Version 2									
m_0, mm	**4,0**	**4,5**	**5,0**	**5,5**	**6,0**	**6,5**	**7,0**	**8,0**	**9,0**
d_{a0}, mm	85	90	95	100	105	110	115	120	125
z_0	14	14	14	12	12	12	12	10	10
K, mm	3,8 (4,0)	4,0	4,2 (4,5)	5,2 (5,0)	5,5	5,7 (5,5)	6,0	7,5	7,8 (8,0)
m_0, mm	**10**	**11**	**12**	**14**	**16**	**18**	**20**	**22**	**25**
d_{a0}, mm	130	150	160	180	200	220	240	250	280
z_0	10	10	9	9	9	9	9	9	9
K, mm	8	9,5	11	12,5	14,0	15,0	16,5	17	19

Helix angles $\gamma_{R,L}$ of both relieved flanks on the pitch cylinder of the right-hand hob are determined by the formula:

$$\mathrm{tg}\,\gamma_R = (p + k\,\mathrm{tg}\,\alpha_{xR})/r_{m0};\quad \mathrm{tg}\,\gamma_L = (p - k\,\mathrm{tg}\,\alpha_{xL})/r_{m0} \quad (4)$$

$r_{m0} = d_{m0}/2$ is the radius of the hob pitch cylinder.
On an arbitrary radius r_i of lateral surfaces, the helix angles γ_i in each point of the profile are determined from expressions:

$$\mathrm{tg}\,\gamma_{Ri} = (p + k\,\mathrm{tg}\,\alpha_{xR})/r_i;\ \mathrm{tg}\,\gamma_{Li} = (p - k\,\mathrm{tg}\,\alpha_{xL})/ri \quad (5)$$

For left-handed hobs, the indices $_{R,\,L}$ in formulas (3)...(5) are reversed.

3. CONTACT LINES WHEN GRINDING HELICAL AND RELIEVED SURFACES

There is the possibility of grinding of helical surfaces of worm threads and relieving of hob flanks by the grinding wheel as initially involute ones at parallel axes O_1-O_1 of the workpiece and O_w-O_w of the grinding wheel with the rectilinear axial profile. The contact line of the wheel and the ground surface touches the base cylinder and intersects the axis of the grinding wheel.

When grinding helical and relieved surfaces with a rectilinear axial profile by a disk grinding wheel, each of the points of the axial profile on the radius r_i of the ground surface with the assigned profile angle α_x and helix angle γ_i can be considered as an element of the involute helical surface. For each point of the axial profile, its base cylinder with radius r_{bi}, can be defined. The angle α_{bi} of the ground surface profile in the plane tangent to the base cylinder is equal to helix angle γ_{bi} of the generatrix and profile angle α_{wi} of the grinding wheel in the same plane. The values of these angles are calculated by a known formula based on the axial angle α_x of the ground surface profile and the helix angle γ_i of the surface in the calculated point of the profile:

$$\mathrm{tg}\,\alpha_{bi} = \mathrm{tg}\,\gamma_{bi} = \mathrm{tg}\alpha_{wi} = (\mathrm{tg}^2\,\alpha_x + \mathrm{tg}^2\,\gamma_i)^{0,5} \qquad (6)$$

We determine the radius r_{bi} of the base cylinder for each point of the axial profile as:

$$r_{bRi} = (p + k\,\mathrm{tg}\,\alpha_{xR})/\mathrm{tg}\,\alpha_{bRi}; \quad r_{bLi} = (p - k\,\mathrm{tg}\,\alpha_{xL})/\mathrm{tg}\,\alpha_{bLi} \qquad (7)$$

We set the axial angle α_x of the profile of the ground helical or relieved surface to be constant. At the section of the teeth adjacent to the hob tooth heads, the helix angle of the ground surface is the smallest for the profile due to the larger diameter, therefore, the angle $\alpha_{ba} = \gamma_{ba} = \alpha_{wa}$ of the profile and inclination of the generatrix of involute surface is the smallest, so here the radius r_{ba} of the base cylinder will be the largest for the ground profile. For the section of tooth roots the similar angle $\alpha_{bf} = \gamma_{bf} = \alpha_{wf}$ is the largest, respectively, the radius r_{bf} of the base cylinder will be the smallest.

Let us refer to Fig. 1. When the axes O_w-O_w of the wheel and O_1-O_1 are parallel, all the tangents to the base cylinders of the points of the tooth profile converge on the axis of the grinding wheel.

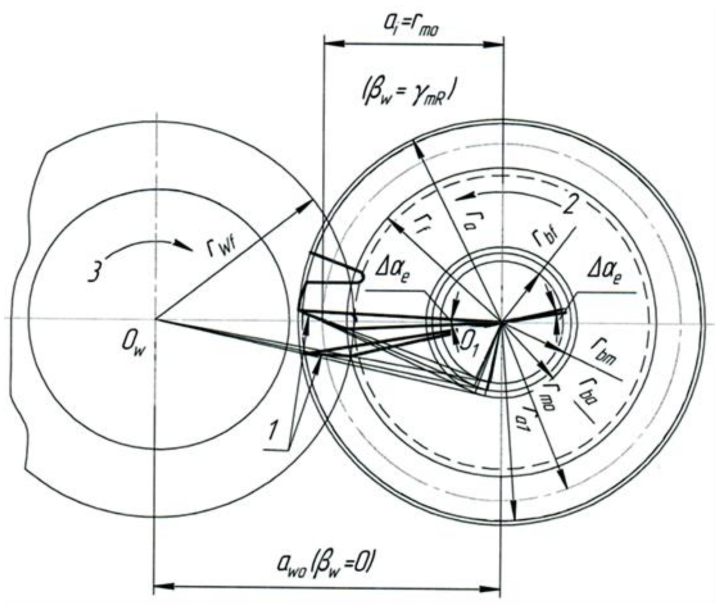

Figure 1: Generation of contact lines when grinding the surface with a rectilinear profile
1 - contact lines of the straight section of the profile, 2 - direction of rotation of the cutter,
3 - direction of rotation of the grinding wheel

The assigned point of the axial profile of the ground surface on the contact line of the wheel surfaces and the workpiece is obtained at the intersection point of a tangent to the base cylinder and the cylinder, on which the assigned point is located. All tangents to base cylinders corresponding to the axial profile points of the ground surface converge in one point of the interaxial perpendicular, and the enveloping line of contact points obtained in this way is the locus of points of common normal lines to these surfaces and it determines the contact line of the surfaces of the wheel and the workpiece.

When the inclination angle β_w of the grinding wheel axis relative to the workpiece axis is changed, tangents to the base cylinders of the ground surface also change their position. Thus, at the value β_w equal to the helix angle of the ground surface on the pitch cylinder, the contact line of the wheel and the ground tooth

surface crosses the interaxial perpendicular at the point on the pitch cylinder, and, thus, all the tangents to base cylinders of the ground surface profile points cross the interaxial perpendicular in the same point.
Obviously, the tangents to base cylinders of the ground surface and the contact line of the latter with the grinding wheel surface may also occupy other positions relative to the interaxial perpendicular, depending on the value of the angle β_w of the grinding wheel axis setting.
And the position of the point of the interaxial perpendicular, where all tangents to base cylinders corresponding to profile points converge, determines the value of the inclination angle β_w of the grinding wheel axis relative to the workpiece axis. If the inclination angle β_w of the wheel axis is equal to the helix angle γ_i of the ground surface within the height of the ground profile, then the intersection of the contact line of the wheel and the ground surface with the interaxial perpendicular takes place within the height of the ground profile at the point on the radius r_i where these angles are equal.
If the inclination angle of the grinding wheel axis differs from the values of angles of elevation of the ground surface, then the tangents to the base cylinders of points of the ground profile intersect with the interaxial perpendicular outside the profile of the ground surface, at a distance a_i from the hob axis. And for a given value of the angle β_w, it should be determined (for example, for the right flank of the teeth of a right-handed hob) from the expression:

$$a_i = (p + k \operatorname{tg} \alpha_{xR}) / \operatorname{tg} \beta_w \tag{8}$$

If the angle β_w is set equal to the pitch helix angle γ_{m0} of the generating surface of the hob, which is defined as $\operatorname{tg} \gamma_{m0} = p / r_{m0}$, then formula (8) will be as:

$$a_m = r_{m0} \, (1 + k \operatorname{tg} \alpha_{xR} / p) \tag{9}$$

The length and configuration of the contact line inside the extreme points of the ground profile with a change in the angle β_w are practically not changed, since the central angle $\Delta\alpha_e$ covering the contact line of the grinding wheel surface and the ground surface is unchanged. Its slope and distance are changed relative to the interaxial perpendicular (Fig. 1).
For the left side of the teeth of the right-handed hob (on the right side of the root, if you look at the front surface of the teeth) in formulas (8), (9) we should replace the sign (+) with the sign (–) and the index (R) with (L) in the notation of the angle α_x.

4. DETERMINATION OF CONTACT LINE COORDINATES

Let us refer to Fig. 2. The initial position of the contact line takes place when the axes of the wheel and hob are parallel, i.e. when $\beta_w = 0$.

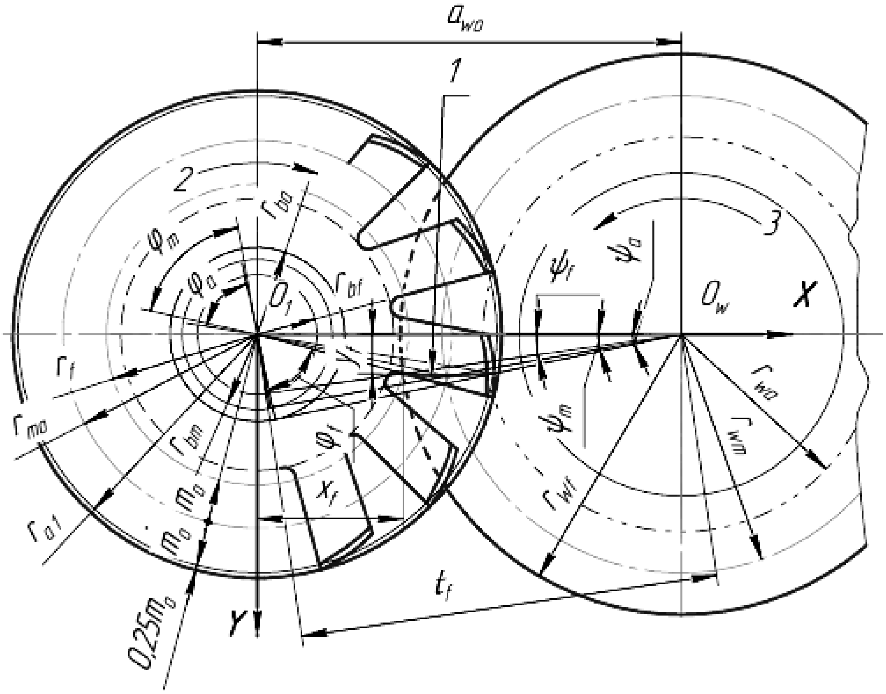

Figure 2: Determination of coordinates of contact lines
1 - contact line, 2 - direction of rotation of the cutter, 3 - direction of rotation of the grinding wheel

This position is remarkable by coincidence and certainty of angles of the ground surface profile and axial profile of the grinding wheel for all points of the ground surface profile due to equality (6) and, therefore, certainty of parameters of the grinding wheel profile curvature, compensating the deviation of the initial involute relieved surface profile from the necessary Archimedean one.
To find coordinates *X, Y* of contact line points, it is first necessary to determine the length a_{w0} of the interaxial perpendicular of wheel and hob axes. It is most convenient to do it with the help of a tangent to the base cylinder of radius r_{bf} of the point of the grinding surface profile on the cylinder of roots of radius r_f, as at this point there is a contact with the grinding wheel on its maximum radius r_{wf} of the active surface. From the axis of the hob, we draw a perpendicular to the tangent of the base cylinder. The tangent passes through the root profile point at radius r_f. The length of the tangent t_f is defined as:

$$t_f = r_{wf} + r_{bf}\ \mathrm{tg}\ \varphi_f \tag{10}$$

where φ_f is the angle between the radii r_{bf} and r_f determined from the expression:

$$\cos \varphi_f = r_{bf} / r_f \tag{11}$$

Similarly, we find the values of the angles φ for the profile points on the head of the hob tooth φ_a and its pitch cylinder φ_m from the expressions:

$$\cos \varphi_a = r_{ba} / r_a; \quad \cos \varphi_m = r_{bm} / r_{m0} \tag{12}$$

The maximum radius r_{wf} of the grinding wheel is taken either from the technical data of the relieving machine: 60 mm for hobs with a diameter d_a up to 160 mm and module up to 12 mm, or 87 mm - for hobs of greater dimensions, or by measuring the current working diameter of the grinding wheel.
Then determine the angle ψ_f between the tangent and the interaxial perpendicular from the expression:

$$\mathrm{tg}\ \psi_f = r_{bf} / t_f \tag{13}$$

and we find the value a_{w0} by formula:

$$a_{w0} = r_{bf} / \sin \psi_f \tag{14}$$

With the value a_{w0} of the grinding wheel and hob interaxial distance found from (14), we find the values of the angles ψ_a, ψ_{m0} between the tangents to base cylinders and the interaxial perpendicular for contact points on the head and pitch cylinders:

$$\psi_a = \mathrm{asin}\ (r_{ba} / a_{w0}), \quad \psi_m = \mathrm{asin}\ (r_{bm} / a_{w0}) \tag{15}$$

We draw the coordinate axes: the origin of axes at the point O_1 of projection of the hob axis, the *X* axis - in the direction of the interaxial perpendicular, the *Y* axis - perpendicular to the *X* axis. Coordinates of reference points of the contact line on cylinders: roots with radius $r_f = r_{m0} - m_0$, pitch with radius r_{m0} and heads with radius $r_{a1} = r_{m0} + m_0$ are defined by expressions:

$$\begin{aligned} X_f &= r_f \sin(\varphi_f + \psi_f); \quad & Y_f &= r_f \cos(\varphi_f + \psi_f); \\ X_m &= r_{m0} \sin(\varphi_m + \psi_m); \quad & Y_m &= r_{m0} \cos(\varphi_m + \psi_m); \\ X_a &= r_{a1} \sin(\varphi_a + \psi_a); \quad & Y_a &= r_{a1} \cos(\varphi_a + \psi_a) \end{aligned} \tag{16}$$

The coordinates of any points within the contact line and, consequently, the length of any section of the contact line can be determined in a similar way.

5. *CALCULATION OF THE GRINDING WHEEL PROFILE ALONG THE CONTACT LINE*

Let us refer to Fig. 3, which shows the axial section of the grinding wheel. At parallel axes of the grinding wheel and hob, knowing from (6) angles $\alpha_{wi} = \alpha_{bi} = \gamma_{bi}$ of the grinding wheel profile in the key points of the grinding surface profile - on cylinders of heads α_{wa}, pitch α_{wm} and roots α_{wf} - you can define radii R_f, R_a of curvature of the axial profile of the grinding wheel.
In the plane tangent to the base cylinder, at the profile point of the hob on its pitch cylinder, the contact point of the wheel profile has the profile angle α_{wm}. This point of the wheel profile is the junction point of the curve sections of radii R_a and R_f. The length of working sections of the wheel profile is defined as m_0 /cos α_{wm}. The radius R_a of the curvature of the ac surface of the grinding wheel machining the hob tooth head is defined as:

$$R_a = (m_0 / \cos \alpha_{wm}) / \sin(\alpha_{wm} - \alpha_{wa}) \tag{17}$$

Radius R_f of curvature of the active surface of the grinding wheel machining the hob tooth root is defined as:

$$R_f = (m_0 / \cos \alpha_{wm}) \,/\, \sin (\alpha_{wf} - \alpha_{wm}) \quad (18)$$

Convexity arrows δ_a and δ_f of the wheel profile in these sections are determined by the formulas:

$$\delta_a = (m_0 / \cos \alpha_{wm})^2 / 2R_a = 0{,}5 m_0 \sin (\alpha_{wm} - \alpha_{wa}) / \cos \alpha_{wm};$$
$$\delta_f = (m_0 / \cos \alpha_{wm})^2 / 2R_f = 0{,}5 m_0 \sin (\alpha_{wf} - \alpha_{wm}) / \cos \alpha_{wm} \quad (19)$$

Thus, at $\beta_w = 0$ (parallel to the axes of the wheel and hob) the contact line defined above and the specified arrows of convexity of the grinding wheel profile compensate the deviation of the axial profile of the involute relieved surface relative to the rectilinear axial profile of the Archimedean relieved surface, which, strictly speaking, is what we aim to obtain.

The radius R_f is somewhat smaller than the radius R_a of the curvature of the section machining the tooth head. Accordingly,

$$\delta_f > \delta_a \quad (20)$$

If the difference in values ($\delta_f - \delta_a$) is within the tooth profile tolerance of ISO 4468:2020 (for a certain hob range), then the axial profile of the wheel can be dressed to a single radius of curvature, preferably to the R_a value. Since in this case the profile of the relieved tooth surface (and the cutting edge) on the hob tooth root closer to the cavity will be more complete (tooth profile deflection in +), and the gear head on the hob will have a small flange. This is in principle acceptable by the standard; and it helps to improve the characteristics of gear meshing in terms of the noise level.

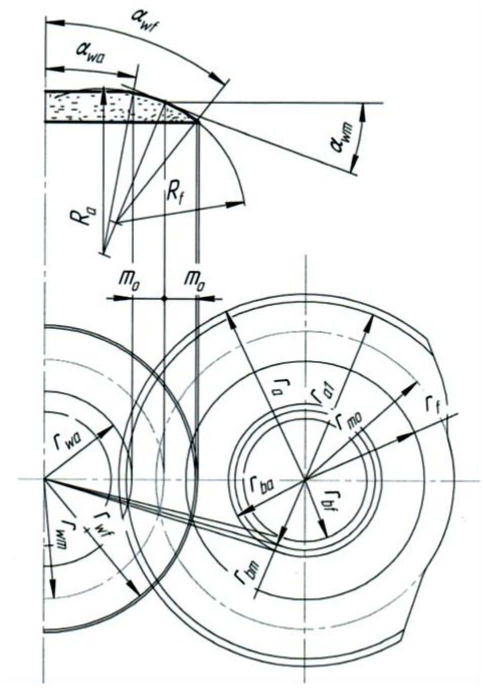

Figure 3: Determining the grinding wheel profile along the contact line

6. CONTACT OF THE GRINDING WHEEL AND TOOTH SURFACES WHEN SETTING THE GRINDING WHEEL AXIS TO ASSIGNED ANGLE

With an arbitrarily assigned angle β_w of the grinding wheel axis, the characteristic of the grinding wheel active surface contact with the relieved surface of the hob teeth is changed. The length of the contact line and its configuration within itself are constant, because the values of the angles φ_i at contact points depend only on the parameters of the relieved surface - the radii of base cylinders and radii of the cylinders of the location of these points. However, when the angle β_w is changed, the contact line rotates relative to the hob axis and shifts relative to the grinding wheel axis. The distance a_i from the hob axis to the intersection point of the tangents to the base cylinders on the interaxial perpendicular does not coincide with the dimension a_{w0}. As a consequence of the above, the angle μ_β is formed between the tangent to the base cylinder of the contact point on the pitch cylinder of the relieved surface and the axial section of the grinding wheel, passing through the same point of the axial profile of the tooth surface (Fig. 4). And to calculate the axial profile of the grinding wheel, this change in position of the contact line of the wheel and the relieved surface should be taken into account.

By formula (8) we determine the value of a_i, then for the key points of the profile ($_{f,\ m,\ a}$) we find:

angles ψ_i between the tangents to the base cylinders and the interaxial perpendicular from the expression

$$\sin \psi_i = r_{bi} / a_i \tag{21}$$

lengths t_i of the tangents to the base cylinders

$$t_i = r_{bi} / \mathrm{tg}\, \psi_i \tag{22}$$

lengths of the segments b_i from the point of intersection of the tangents with the interaxial perpendicular to the corresponding point of the ground surface profile:

$$b_i = t_i - r_{bi} / \mathrm{tg}\, \varphi_i \tag{23}$$

and then you can calculate the coordinates X_i, Y_i of points of the contact line by the formulas:

$$X_i = a_i - b_i \cos \psi_i; \qquad Y_i = b_i \sin \psi_i \tag{24}$$

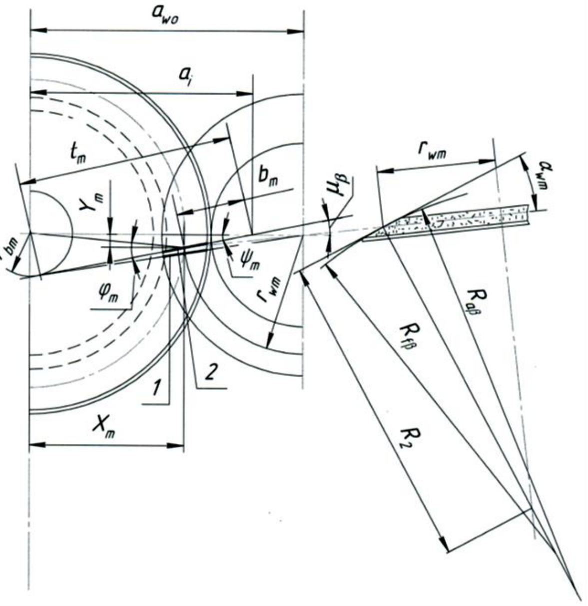

Figure 4: Determination of the contact line and axial profile of the grinding wheel at $\beta_w \neq 0$
1 – contact line at aw0, $\beta w = 0$; 2 – contact line at ai, $\beta w \neq 0$

In general case we refer parameters R_a, δ_a, R_f, δ_f of curvature of the grinding wheel profile defined by formulas (17) ... (19) to the normal section of the grinding wheel surface in the plane, tangent to the base cylinder of the profile point on the pitch cylinder, as the contact point on the pitch cylinder is a junction point of two sections of the grinding wheel profiling. The value of the angle μ_β of the deviation of the plane, in which the curvature parameters of the grinding wheel are defined, from the axial section of the grinding wheel will be determined from the relation:

$$\mu_\beta = \mathrm{acos}\, (Y_m / r_{wm}) - (90° - \psi_m) \tag{25}$$

where $r_{wm} = (r_{wf} - m_0)$ is the radius of the grinding wheel at the contact point on the pitch cylinder of the hob.

To determine the parameters of the axial profile curvature of the grinding wheel at an arbitrarily assigned angle β_w we will use the Euler formula, which describes the ratio of curvatures of lines on the surface: for each normal section of the surface its curvature $1/R$ is defined as[9 - 11]:

$$1/R = (\cos^2\mu / R_1) + (\sin^2\mu / R_2) \tag{26}$$

where R_1 and R_2 are the basic radii of curvature, i.e. *the largest and the smallest* values of R; they are obtained at the basic normal sections [11];

μ is the angle between the planes of normal sections with the basic radius of curvature R_1 and the current radius of curvature R; in this case, it is between the axial section of the grinding wheel and the section tangent to the base cylinder, where the radii of curvature are defined by formulas (17) and (18).

$$\mathrm{tg}\, \mu = \mathrm{tg}\, \mu_\beta \cos \alpha_{bm} / \cos \beta_w \tag{27}$$

With respect to the question under consideration, in formula (26) R are the radii R_a and R_f of the curvature sections in the defined (25) normal section to the surface of the grinding wheel along the contact line found by formulas (17) and (18); R_1 are the sought radii of curvature of the axial section of the wheel, corresponding to the profiling sections for the heads and roots of teeth; R_2 is the radius of curvature of the grinding wheel in the plane perpendicular to the axial profile of the grinding wheel at the contact point on the hob pitch cylinder. R_2 is determined by the formula:

$$R_2 = r_{wm} / \sin \alpha_{wm} \tag{28}$$

The value of the angle α_{wm} of the grinding wheel profile in its axial section at the contact point with the hob pitch cylinder is determined from the relation:

$$\mathrm{tg}\, \alpha_{wm} = \mathrm{tg}\, \alpha_{bm} \cos\mu \tag{29}$$

The values of sought radii $R_{a\beta}$ and $R_{f\beta}$ of curvature (as R_1) in the axial section of the grinding wheel are obtained from (26):

$$R_1 = R_{a,f}\, R_2 \cos^2\mu \,/\, (R_2 - R_{a,f} \sin^2\mu) \tag{30}$$

Our experience showed that when grinding the face of the hobs of the most widespread type Version 2, for the flank of these hobs where the helix angle of the generating surface is greater than the helix angle of the relieved surface (for example, the left flank of the hob teeth with the right direction of the generating surface), the accuracy of the profile of the generating rack is ensured with a multiple margin for all the hob types when grinding with a straight profile, and the contact line is also practically straight. For the opposite flanks (right flank of the tooth of a right-handed hob) in hobs with modules up to 2.25 mm, the accuracy of the flank profile is also technologically ensured according to accuracy class 4A without the use of radius dressing. With increasing the module, depending on the required accuracy class of the hob, radius dressing of the grinding wheel profile is required according to the above method.

7. DETERMINATION OF A SINGLE ANGLE OF INCLINATION OF THE GRINDING WHEEL AXIS

Setting of the grinding wheel axis inclination angle, which is uniform for grinding both flanks, provides reducing the machine readjustment to the change of grinding wheel on the machine spindle and thus limiting the time of readjustment only by achieving the required profile of the grinding wheel active surface. In addition, when using machines with a dressing mechanism that allows profiling both sides of the grinding wheel, such a setting will make it possible to produce a double-sided relieving of teeth of worm hobs.

The conditions for a single angle of the grinding wheel setting for grinding both relieved surfaces are: firstly, the intersection of the grinding wheel axis by normal lines to these surfaces; secondly, a single starting point for grinding both flanks of the teeth.

The beginning of tooth machining on each flank takes place at the apex of the tooth, since it is the apex of the tooth that initially crosses the contact line of surfaces of the hob tooth and the grinding wheel. In addition, on the side where the helix angle is greater than that of the generating surface of the hob (on right flanks of right-handed hobs), due to greater values of the radii of the base cylinders of profile points, there is a more extended area of tooth entry to the full grinding height and, accordingly, the length of the relieving stroke and the arc of hob rotation during this stroke. In this regard, a single starting point for both flanks as a necessary condition should be defined by crossing the contact lines of the right and left flanks not on the outer diameter of the straight section of the tooth profile, but at the point near the pitch cylinder, where there is the middle of the infeed section on both flanks; and where the helix angles of the generating surface of the hob and the front surface of teeth coincide.

7.1 Determining the point of intersection of contact lines

The intersection of contact lines of flanks on the pitch cylinder is defined as the intersection of the tangents to the base cylinders at the profile points on the pitch cylinder.

In accordance with ISO 2490:2007, the relieved surfaces of hob teeth with modules to 11 mm have different directions (right and left), the distances a_{wR}, a_{wL} from the hob axis to the points of intersection of tangents to the base cylinders with the interaxial perpendicular have different signs, and tangents to base cylinders are on the same side of the interaxial perpendicular (Fig. 5).

The relieved surfaces of teeth of hobs with modules of 12 mm and more have one direction - right for right-handed hobs; the distances a_{wR}, a_{wL} have the same signs (+), the beginning of tangents to the base cylinders are on different sides of the interaxial perpendicular (Fig. 6), and the intersection point of these tangents is on the side which refers to the base cylinder with a greater diameter.

The point of intersection of the tangents to both base cylinders of the profile points on the pitch cylinder of the rectilinear part of cutting edges is found in general form as follows:

1. The value of the angle β_{w2} is assigned. The index ($_2$) denotes a single angle for both flanks.

2. The distances a_{wR}, a_{wL} from the hob axis to the points of intersection of tangents to the base cylinders of points of profiles with the interaxial perpendicular are determined by expressions:

$$a_{wR} = (p + k \text{ tg } \alpha_{xR}) / \text{tg } \beta_{w2}; \quad a_{wL} = (p - k \text{ tg } \alpha_{xL}) / \text{tg } \beta_{w2} \tag{31}$$

3. The angles between the tangents and the interaxial perpendicular (ε_R for the right flank, ε_L for the left flank) are determined from the expressions:

$$\sin \varepsilon_R = r_{bRm} / a_{wR}; \quad \sin \varepsilon_L = r_{bLm} / a_{wL} \tag{32}$$

4. The distance Y_{c1} from the interaxial perpendicular to the point of intersection of the contact lines of the profiles is determined from the geometric constructions by the expression:

$$Y_c = (a_{wR} - a_{wL}) \text{ tg } \varepsilon_R \text{ tg } \varepsilon_L / (\text{tg } \varepsilon_R + \text{tg } \varepsilon_L) \tag{33}$$

5. We find the radius r_{yc1} of the hob at which the point of intersection of the tangent projections and, respectively, the contact lines of the profiles is located:

$$r_{yc} = [(Y_c / \text{tg } \varepsilon_L) + a_{wL})^2 + Y_c^2]^{0,5} \tag{34}$$

and compare its value with r_m.
The values r_{bLm} and a_{wL} in the above formulas are taken with the corresponding sign from (7) and (31).

7.2 Calculating the angle of inclination of the grinding wheel axis

The initial value of the angle β_{w2-1} of the grinding wheel axis setting for relieving the corresponding dimension type of the hob is determined from (2), equal to the helix angle γ_{m0} of the generating surface on the pitch cylinder.
The necessary search of the setting begins with calculation of coordinates Y_c and r_{yc} of the intersection point of contact lines by formulas (31 ... 34). The values of radii r_{bRm}, r_{bLm} of the base cylinders for profile points on the pitch cylinder should be determined by formula (7). If the obtained value of radius r_{yc1} does not coincide with the value of radius r_m of the pitch cylinder, then we find the parameter p_{rc} of change of radius r_{yc1} depending on the angle β_{w2} in the form:

$$p_{rc1} = r_{yc1} \text{ tg } \beta_{w2-1} \tag{35}$$

The index ($_1$) in formula (35) indicates the order of approximation to the sought value of r_{yc}. Next, we find the following approximation of the angle β_{wc2-2} of the inclination of the grinding wheel axis to get the point of intersection of the contact lines on the hob pitch cylinder:

$$\text{tg } \beta_{wc2-2} = p_{rc1} / r_m \tag{36}$$

And again, we repeat the calculation by (32) ... (34) of coordinates of the point of intersection of tangents to the base cylinders of the points of ground surface profile on the pitch cylinder. If in this case it is also $r_{yc2} \neq r_m$, we again specify the parameter p_{rc} of radius similarly to (35): $p_{rc2} = r_{yc2} \text{ tg } \beta_{wc2-2}$ and determine a new approximation of the angle of the wheel axis setting similarly to (36): $\text{tg } \beta_{wc2-3} = p_{rc2} / r_m$.
Calculations stop at achieving the approach of values r_{yc} and r_m within limits ±0,1mm. Thus, by method of consecutive approximations we define the required angle β_{w2} of the grinding wheel axis installation, at which almost simultaneous entrance in machining of both flanks is provided.
Fig. 5 shows the search for the point *C* of intersection of contact lines on the example of a single-thread right-handed hob with the module of 4 mm. Table 2 shows the calculated parameters of successive approximation to the required result - the location of the intersection point of tangents to the base profile cylinders on the pitch cylinder of this hob. The initial data: m_0 = 4 mm, Z = 1, d_{a0} = 85 mm; the design data: r_m = 37,5 mm, γ_{m0} = 3,0643°, p = 2,003 mm, k = 8,467 mm, α_{Rx} = 20,107°, α_{Lx} = 19,934°, γ_{Rm} = 7,7487°, γ_{Lm} = –1,631 °, α_{bRm} = 21,333°, α_{bLm} = 19,990°, r_{bRm} = 13,0654 mm, r_{bLm} = – 2,935 mm.

Table 2: Example of calculating the setting angle β_{w2} of the axis of the grinding wheel for relieving hobs with the ratio of parameters $p < k$ tg α_{x0}

№ of approximation	β_{w2}, °	a_{wR}, mm	a_{wL}, mm	ε_R, °	ε_L, °	Y_c, mm	r_{yc}, mm	p_{rc}, mm	$\beta_{w2-1,2,3}$, °
1	3,0643	95,318	–19,945	7,8784	8,4622	8,264	36,546	1,9567	2,9868
2	2,9868	97,7937	–20,463	7,6778	8,2465	8,2595	37,445		
By the machine scale	3,0	97,365	–20,373	7,7118	8,2831	8,2602	37,292		

During real machining, the calculated value β_{w2}, at which the value $r_{yc} = r_m$ is achieved, is assigned on the relieving machine with the accuracy of the reference scale for setting the angle of inclination of the grinding wheel axis. For older models of machines, the accuracy of setting for this parameter was 0,1°. That is, in the case considered, this relieving machine should be set to β_{w2} = 3,0°. The radius of the point of intersection of the contact lines will be ≈37.3 mm. The deviation will be 5% of the module.

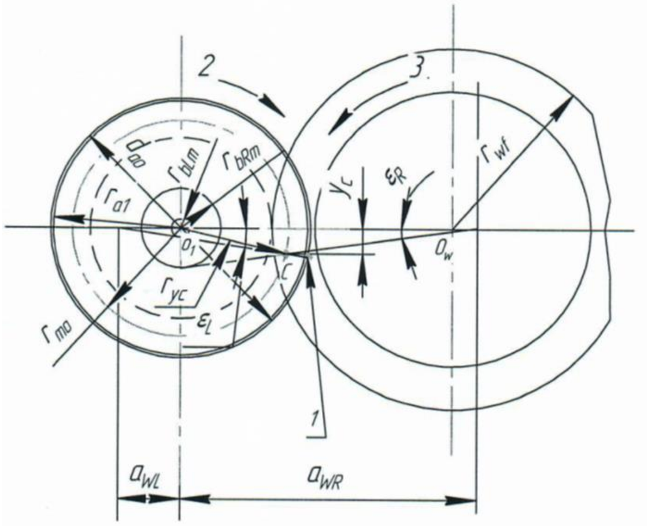

Figure 5: Determination of the angle β_{w2} when relieving the flanks with different directions (m_0 = 4 mm)
1 - the top of the cutter tooth, 2 - the direction of rotation of the cutter,
3 - the direction of rotation of the grinding wheel

For hobs with the parameter ratio $p > k$ tg α_{0x} the method of calculation of coordinates of the point *C* of intersection of the contact lines remains the same, using the parameter p_{rc} and the contact radius r_{yc}, as it is shown in (35) and (36). Fig. 6 shows the location of point *C* when relieving the right-handed hob with a module of 16 mm.

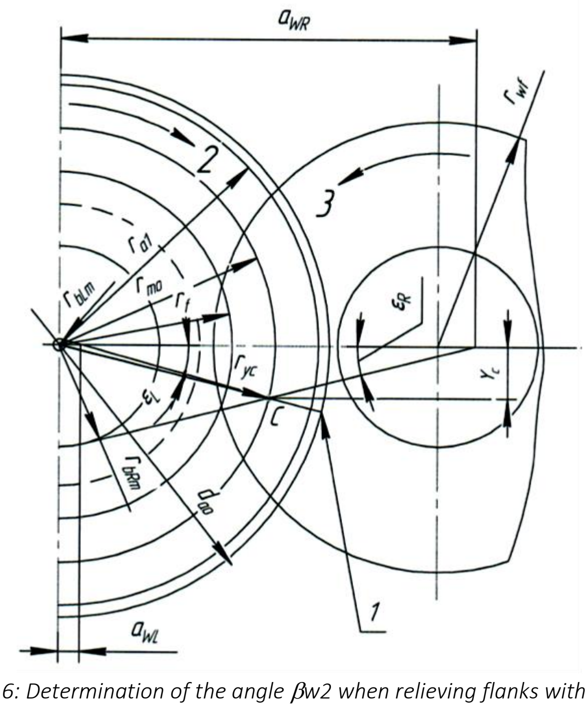

Figure 6: Determination of the angle $\beta w2$ when relieving flanks with one direction (m_0 = 16 mm)
1 - the top of the cutter tooth, 2 - the direction of rotation of the cutter,
3 - the direction of rotation of the grinding wheel

Table 3 shows a sequential approximation of the radius of the point location to the radius of the pitch cylinder with the calculation of the value of the inclination angle of the grinding wheel axis when relieving this hob. The initial data: m_0 = 16 mm, Z = 1, d_{a0} = 200 mm; the design data: r_m = 80 mm, γ_{m0} = 5,739°, p = 8,04 mm, k = 20,0535 mm, α_{Rx} = 20,26°, α_{Lx} = 19,923°, γ_{Rm} = 10,925°, γ_{Lm} = 0,553°, α_{bRm} = 22,614°, α_{bLm} = 19,929°, r_{bRm} = 37,072 mm, r_{bLm} = 2,13 mm.

Table 3: Example of calculation of the setting angle β_{w2} of the grinding wheel axis for relieving of hobs with the ratio of parameters $p > k\ tg\ \alpha_{x0}$

№ of approximation	β_{w2}, °	a_{wR}, mm	a_{wL}, mm	ε_R, °	ε_L, °	Y_c, mm	r_{yc}, mm	p_{rc}, mm	$\beta_{w2-1,2,3...}$ °
1	5,739	153,654	7,682	13,961	16,096	19,495	77,725	7,811	5,577
2	5,577	158,153	7,907	13,557	15,627	19,456	79,87		
By the machine scale	5,6	157,494	7,8745	13,614	15,694	19,461	79,556		

When relieving on a machine with a vernier accuracy of 0.1° the reference scale for the angle β_{w2} should be set to β_{w2} = 5,6°. The radius of the location of the intersection point of the contact lines will be 79.556 mm - the deviation from the pitch cylinder is 0.444 mm, which in this case is negligibly small – 2.8% of the module.
Obviously, by creating machines with a dressing mechanism that allows both sides of the grinding wheel to be profiled simultaneously, this setup will allow for double-sided relieving of hob teeth.

8. CONCLUSION

A method of constructing the contact lines between grinding wheel surfaces and ground flanks of hob teeth on the basis of considering each of the points of axial profile of the relieved surface as an involute helical surface element has been found.
It is determined that all the tangents to the base cylinders corresponding to the axial profile points of the ground surface, converge at one point of the interaxial perpendicular, and the enveloping line of the contact points thus obtained is the locus of the points of common normal lines to these surfaces and determines the contact line of the grinding wheel surfaces and the hob teeth. A procedure for calculating the coordinates of contact line points is given.
The length and configuration of the contact line inside the extreme points of the ground profile with changing the inclination angle β_w of the grinding wheel axis is practically not changed, as the central angle covering the contact line of the grinding wheel surface and the ground surface is unchanged. Its inclination and distance relative to the interaxial perpendicular is changed.
A procedure for calculating the grinding wheel profile for obtaining a rectilinear profile of the relieved surface is developed, including the case when the contact line does not coincide with the axial section of the grinding wheel.
The method of calculating the inclination angle of the grinding wheel axis, which is uniform for adjustment of grinding both flanks, is determined. The value of this angle for a number of dimension types of hobs differs from the generally accepted one, equal to the helix angle of the generating surface on the pitch cylinder.

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