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Multi-Criteria Decision Making Methods: Comparative Analysis of PROMETHEE and VIKOR

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Abstract

In multiple criteria decision making process there are numerous different approaches. For example, Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) is an outranking method, while VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is a compromise ranking method. A comparative analysis of these two methods is presented in this paper. Both methods were applied for selection of energy supply system for space heating of the residential building. Ranking of alternatives using these two methods is different in this particular case. This implies that based on the approach and method used, results may vary. Decision makers should take this into consideration when choosing a method for solving multi-criteria decision making problems.

Key words: *Comparison, Decision making, PROMETHEE, VIKOR*

1. INTRODUCTION

Multiple criteria decision making (MCDM) is the process of finding best option from all of the feasible alternatives in the presence of multiple, usually conflicting criteria [1]. Since criteria are often conflicting there may be no solution satisfying all criteria simultaneously. Consequently, result of this process is often a compromise solution, which is obtained according to decision makers' preferences. Main steps of MCDM are defined as follows [2]:

- Establishing system evaluation criteria that relate system capabilities to goals
- Developing alternative systems for attaining the goals
- Evaluating alternatives in terms of criteria
- Applying a normative multi-criteria analysis method
- Accepting one alternative as ''optimal'' (preferred)
- If the final solution is not accepted, gather new information and go into the next iteration of multi-criteria optimization.

Wide range of methods can be applied for solving MCDM problems. The problem of selecting appropriate MCDM method has been addressed in literature [3] [4] [5]. Nevertheless, there is no perfect solution for this problem, due to diversified approach of different methods. Often, different methods can give us different ranking of alternatives. The inconsistency in results occurs because [1]:

- Algorithm for selection of preferred alternative differs
- Techniques use criteria weights differently in their calculations
- Algorithms attempt to scale the objectives, thus affecting the weights already chosen
- Algorithms introduce additional parameters that affect selection of preferred alternative.

For this reason there were some attempts to find similarities and differences among MCDA methods in literature. Eight different methods were compared in [1], while VIKOR and TOPSIS were compared in [2]. VIKOR, TOPSIS and SAW were compared in [6]. On the basis of a case study, outranking methods are compared to techniques based on the ideas of multi attribute utility theory [7]. Opricovic and Tzeng [8] compared VIKOR with TOPSIS, ELECTRE and PROMETHEE. VIKOR, TOPSIS, ELECTRE and PROMETHEE were compared in [9] as well.

In this paper, we will compare PROMETHEE and VIKOR on an empirical example. We used these MCDA methods for selection of energy supply system for space heating of the residential building. We chose to compare these two methods because of the difference in their approach to problem solving situation. PROMETHEE is an outranking method, while VIKOR is a compromise ranking method.

The reminder of the paper is structured as follows. In Section 2 procedures of PROMETHEE and VIKOR are explained. In Section 3 these two methods are applied on an empirical example. Section 4 presents discussion and conclusion of this paper.

2. MCDM METHODS

In this section we described PROMETHEE and VIKOR methods, as a theoretical basis for the following application.

2.1 PROMETHEE Method

The PROMETHEE family of outranking methods, including the PROMETHEE I for partial ranking of the alternatives and the PROMETHEE II for complete ranking of the alternatives, were developed by Brans [10]. PROMETHEE II is described in this part of the paper, since majority of researchers have referred to this version of the method [11]. This method is based on a pairwise comparison of alternatives in respect to each defined criterion. The implementation of PROMETHEE II requires two types of information. Decision maker needs to define weight and preference function for each criterion. Weight determines the importance of each criterion, while preference function serves to translate difference between the evaluations obtained by alternatives into a preference degree ranging from zero to one. Vinke and Brans [12] proposed six types of preference functions: (1) usual criterion, (2) U-shape criterion, (3) V-shape criterion, (4) level criterion, (5) V-shape with indifference criterion and (6) Gaussian criterion. The procedure of PROMETHEE II method is as follows [10]:

Step 1: Determination of deviations based on pairwise comparisons

$$d_j(a,b) = g_j(a) - g_j(b) \tag{1}$$

Where $d_j(a,b)$ denotes the difference between the evaluations of a and b on each criterion.

Step 2: Application of the preference function

$$P_j(a,b) = F_j\left[d_j(a,b)\right] \qquad j = 1,\dots,k \tag{2}$$

Where $P_j(a,b)$ denotes the preference of alternative a with regard the alternative b on each criterion, as a function of $d_j(a,b)$.

Step 3: Calculation of an overall or global preference index

$$\forall\, a,b \in A, \qquad \pi(a,b) = \sum_{j=1}^{k} P_j(a,b)\, w_j \tag{3}$$

Where $\pi(a,b)$ of a over b (from 0 to 1) is defined as a weighted sum $p(a,b)$ of each criterion, and w_j is the weight associated with the expressing the decision maker's preference as the relative importance of the *j*-th criterion.

Step 4: Calculation of outranking flows

$$\phi^+(a) = \frac{1}{n-1}\sum_{x\in A}\pi(a,x) \tag{4}$$

$$\phi^-(a) = \frac{1}{n-1}\sum_{x\in A}\pi(x,a) \tag{5}$$

Where $\phi^+(a)$ and $\phi^-(a)$ denote the positive outranking flow and negative outranking flow for each alternative, respectively.

Step 5: Calculation of net outranking flow

$$\phi(a) = \phi^+(a) - \phi^-(a) \tag{6}$$

Step 6: Determine the ranking of all the considered alternatives depending on the values of $\phi(a)$. Higher value of $\phi(a)$, means better ranking of the alternative. Thus, the best alternative is the one having the highest $\phi(a)$ value.

2.2 VIKOR Method

VIKOR is a MCDA method introduced by Opricovic [13]. It determines the compromise ranking list and the compromise solution. Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The compromise ranking algorithm VIKOR has the following steps [2]:

Step 1: Determine the best f_i^* and the worst f_i^- values of all criterion functions, *i* = 1,2,…,n.
If the *i*-th function represents a benefit, then:

$$f_i^* = \max_j f_{ij}, \qquad f_i^- = \min_j f_{ij}$$

If the *i*th function represents a cost, then:

$$f_i^* = \min_j f_{ij}, \qquad f_i^- = \max_j f_{ij}$$

Step 2: Compute the values S_j and R_j, *j* = 1,2,…,J, by the relations

$$S_j = \sum_{i=1}^{n} w_i\left(f_i^* - f_{ij}\right)/\left(f_i^* - f_i^-\right) \tag{7}$$

$$R_j = \max_i\left[w_i\left(f_i^* - f_{ij}\right)/\left(f_i^* - f_i^-\right)\right] \tag{8}$$

where w_i are the weights of criteria, expressing their relative importance for the decision maker.

Step 3: Compute the values , *j* = 1,2,…,J, by the relation

$$Q_j = \frac{v\left(S_j - S^*\right)}{\left(S^- - S^*\right)} + \frac{(1-v)\left(R_j - R^*\right)}{\left(R^- - R^*\right)} \tag{9}$$

where

$$S^* = \min_j S_j, \qquad S^- = \max_j S_j$$

$$R^* = \min_j R_j, \qquad R^- = \max_j R_j$$

and v is introduced as weight of the strategy of the maximum group utility, whereas $1 - v$ is the weight of the individual regret.

Step 4: Rank the alternatives, sorting by the values S, R and Q, in decreasing order. The results are three ranking lists.

Step 5: Propose as a compromise solution the alternative (a') which is ranked the best by the measure Q (minimum) if the following two conditions are satisfied:

C1: "Acceptable advantage"

$$Q(a'') - Q(a') \geq DQ$$

where a'' is the alternative in the second position in the ranking list by Q; $DQ = 1/(J-1)$; J is the number of alternatives.

C2: "Acceptable stability in decision making":
Alternative (a') must also be the best ranked by S or/and R.
If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives a' and a'' if only condition C2 is not satisfied, or
- Alternatives a', a'',..., $a^{(M)}$ if condition C1 is not satisfied; and $a^{(M)}$ is determined by the relation $Q\left(a^{(M)}\right) - Q(a') < DQ$ for maximum *M* (the positions of these alternatives are "in closeness").

3. COMPARING PROMETHEE II AND VIKOR ON AN ILLUSTRATIVE EXAMPLE

In this section we will introduce example on which we will apply PROMETHEE II and VIKOR. After that, we will present results obtained by these two methods.

3.1 **Illustrative Example**

For the purpose of this paper, selection of energy supply system for space heating of the residential building will be performed. There are several different energy supply systems for space heating of the residential building that could be used. Natural gas boiler, pellet boiler, coal boiler and heat pump are considered in this case. Those alternatives are compared on several different and conflicting criteria. Literature review on the application of the MCDM techniques to the energy issues shows that evaluation criteria for alternative energy sources can be grouped into four main categories: technical, economic, environmental, and social [14]. Selection of criteria that are used in this example is based on the work presented in [15]. We chose the most used criteria from previous research in this area, namely: investment costs, annual expenses, CO_2 emissions, efficiency, and comfort of the end users. All relevant information, necessary for the use of MCDM methods, is presented in Table 1.

Table 1. Parameters for multi-criteria decision analysis

	Criteria	Investment	Annual expenses	CO_2 emissions	COP	Comfort
	Unit	€	€	kg/a	/	5-point
	Min/Max	Min	Min	Min	Max	Max
	Weight	0,30	0,25	0,20	0,10	0,15
Alternatives	Gas boiler	3.500,00	3.850,00	20143	0,90	Very good
	Pellet boiler	3.800,00	3.000,00	500	0,85	Good
	Coal boiler	2.900,00	2.900,00	48644	0,75	Average
	Heat pump	7.400,00	1.950,00	10957	4	Very good

3.2 **Results**

In this section we will present results obtained by PROMETHEE II and VIKOR.

3.2.1 *PROMETHEE II Results*

Complete ranking of the alternatives when PROMETHEE II is used for evaluation is presented in Table 2. Preference functions which were used in this case are: V-shape with indifference criterion was used for comparison of alternatives in regards to investment costs and annual expenses; V-shape criterion was used for comparison of alternatives in regards to CO_2 emissions and efficiency; Level criterion was used for comparison of alternatives in regards to comfort of the end user.

Table 2. PROMETHEE II complete ranking of alternatives

	ϕ	ϕ^+	ϕ^-	Rank
Heat pump	0.1250	0.4182	0.2933	1
Pellet boiler	0.1237	0.2527	0.1291	2
Gas boiler	-0.0099	0.2193	0.2293	3
Coal boiler	-0.2387	0.1368	0.3755	4

Ranking of alternatives is as follows: heat pump is the most preferred energy supply system (0.1250), followed by pellet boiler (0.1237), gas boiler (-0.0099), and coal boiler (-0.2387). Renewable energy sources such as heat pump and pellet boiler are better ranked then non-renewable energy sources such as gas and coal boilers.

3.2.2 *VIKOR Results*

Ranking of the alternatives when VIKOR is used for evaluation is presented in Table 3.

Table 3. VIKOR complete ranking of alternatives

	S_j	R_j	Q_j	Rank
Pellet boiler	0.3701	0.1382	0.0576	1
Heat pump	0.3434	0.3000	0.5000	2
Gas boiler	0.4669	0.2500	0.6121	3
Coal boiler	0.5750	0.2000	0.6910	4

Ranking of alternatives is as follows: pellet boiler is the most preferred energy supply system (0.0576), followed by heat pump (0.500), gas boiler (0.6121), and coal boiler (0.6910). In this case, renewable energy sources such as pellet boiler and heat pump are better ranked then non-renewable energy sources such as gas and coal boilers, as well. The only difference is in reverse order of pellet boiler and heat pump compared to results of PROMETHEE II method.

4. DISCUSSION AND CONCLUSION

From results presented in Tables 2 and 3 we can see that they vary based on the method used. PROMETHEE II gives the following order of alternatives: heat pump, pellet boiler, gas boiler, and coal boiler. VIKOR rankings are slightly different: pellet boiler, heat pump, gas boiler, and coal boiler. If we analyse this in a broader sense, than results of both methods favour renewable over non-renewable energy sources. The difference is in order of two renewable sources. PROMETHEE II favours heat pump over pellet boiler. However, these two alternatives are pretty close to each other. VIKOR favours pellet boiler, and based on Q values this alternative is much better than all the others.

This difference in results can be explained with different approach to MCDM problems of these two methods. Opricovic [8] argue that PROMETHEE gives the results that are in compliance with S value in VIKOR, which presents the maximum group utility. This is confirmed in this case, as well. VIKOR additionally uses R values, which represent minimum individual regret, to evaluate alternatives. This is the main reason for different results of these methods in this particular case. Based on R values, pellet boiler is the best alternative while heat pump is the worst. The results of PROMETHEE II and VIKOR are different in [9] as well.

From results and analysis performed in this paper we can confirm that PROMETHEE II and VIKOR can give different results when applied to the same problem. This does not necessary mean that one approach is better than the other. It simply implies that it is important for decision makers to get familiar with different MCDA methods so they can apply appropriate method for problem solving based on their preferences.

Our analysis and conclusion related to PROMETHEE II and VIKOR is limited only to the particular example given in this paper. More research in this direction should be performed in future. Better understanding of different MCDA methods could help decision makers to make more reliable decisions.

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